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Load to Motor Inertia Mismatch: Unveiling The Truth

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Introduction

Servo system inertia mismatch, between load and motor, has long been a concern for the motion system designer. Component suppliers often have their “rules of thumb” for making recommendations, but often times these rules vary dramatically from one manufacturer to the next, adding to the confusion.

This paper addresses the issue of inertia mismatch from a theoretical and practical standpoint. Fundamentally, power transmission is optimized in a mechanical system if the load inertia matches the motor inertia. This may, or may not be practically implemented with gearing between motor and load. Gearing inertia, inefficiency, mechanical limits, and cost all factor into the decision as to whether power transfer should be optimized. From a practical viewpoint, large inertia mismatches between load and motor can play havoc on the stable operation of a servo system. This is most pronounced when there is lost motion or compliance between motor and load. Consideration must be made for the type of mechanism being driven, and the practical allowable limit for inertia mismatch. For example, the tolerable limit of inertia mismatch is much higher for a direct driven stiffly coupled load compared to that of a load coupled by a belt or chain drive.

Derivations of the theoretical optimization, and practical guidelines for typical mechanisms, are provided to dispel the misunderstanding and confusion frequently associated with this subject.

The Basics of Acceleration

Newtonian physics teaches us that F(force) = M(mass) x A(acceleration), or in rotary terms, T(torque) = J(inertia) x A(acceleration). This fundamental equation shows us that the less inertia a system has, the less torque it will take to meet a desired acceleration rate. For this reason it is advantageous to minimize inertia to the greatest extent to maximize acceleration. For a fixed amount of load inertia this means minimizing motor inertia. Stated another way, minimizing motor inertia would allow most of the motor’s torque to accelerate the load not ‘wasting’ much of the motor’s torque accelerating its own inertia. In conclusion, minimizing motor inertia for a given rating of torque will theoretically maximize acceleration, increase system bandwidth, but at the same time, increases load to motor inertia mismatch.

Maximum Power Transfer For Maximum Acceleration

Maximum power transfer will occur in a mechanical system if the inertia of the load matches the inertia of the motor. That is, for a specific motor, if load inertia

reflected to the motor shaft can be made to match the motor inertia, disregarding added inertia and inefficiency of the reducer, power transfer will be optimized and maximum acceleration of the load will result. The proof for this is detailed, concluding with Equation 1. This concept is somewhat intuitive in consideration of some basic concepts of the application of gearing between motor and load as indicated in Table 1.
The following relationships apply :

$\text{Output Speed} = \dfrac{\text{Input Speed}}{\text{Gear Ratio}}$ $\text{Where: Gear Ratio} = \dfrac{\text{Turns of the Motor}}{\text{Turns of the Load}}$
$\text{Output Torque} = \text{Input Torque x Gear Ratio x Reducer Efficiency}$ $\text{Where: Reducer Effeciency} = \dfrac{\text{\% of Efficiency}}{100}$
$\text{Reflected Load Inertia} = \dfrac{\text{Load Inertia}}{\text{Reducer Ratio}^2} + \text{Reducer Inertia}$ Where: Reflected Load Inertia is the equivalent inertia of the load seen by the motor

TABLE 1- Basic Gearbox Relationships

Reflected load inertia is reduced by the square of a reducer ratio, while speed is only increased by the ratio. This suggests that for systems having a large load inertia relative to the motor, motor torque can be reduced by the use of a reducer because the torque required to accelerate the load inertia is reduced by the square of the ratio, while only increasing the torque required to accelerate the motor by the ratio (speed increase). On the other hand, if the load inertia is small, increasing the gear ratio will have minimal effect on the total system inertia while increasing the torque required for acceleration of the motor itself.

Load Inertia Optimization Proof

Assumptions:

1) Reducer inefficiency and friction will be ignored
2) Gear or pulley inertia directly coupled to the motor shaft will be considered part of the motor inertia
3) Gear or pulley inertia coupled to the load shaft will be considered part of the load inertia

Torque required at the motor is:

$$T_M = \left(J_M + \frac{J_L}{Gr^2}\right) x\ \alpha_M$$

Where:

T_M = Motor Torque

J_M = Motor Inertia

J_L = Load Inertia

Gr = Gear Ratio

α_M = Acceleration of motor

Acceleration at the motor is:

$$\alpha_M = \alpha_L\ x\ Gr$$

Where:

α_L = Acceleration of load

Combining and rearranging yields:

$$\alpha_L = \frac{T_M\ x\ Gr}{J_M\ x\ Gr^2 + J_L}$$

Taking the derivative of α_L with respect to Gr results in:

$$\frac{d\alpha_L}{dGr} = \frac{\left(J_M Gr^2 + J_L\right)\left(T_M Gr\right)' - \left(T_M Gr\right)\left(J_M Gr^2 + J_L\right)'}{\left(J_M Gr^2 + J_L\right)^2}$$

$$\frac{d\alpha_L}{dGr} = \frac{\left(J_M Gr^2 + J_L\right)T_M - T_M Gr\left(2J_M Gr\right)}{\left(J_M Gr^2 + J_L\right)^2}$$

Rearranging yields:

$$\frac{d\alpha_L}{dGr} = \frac{T_M\left(J_L - J_M Gr^2\right)}{\left(J_M Gr^2 + J_L\right)^2}$$

Setting the derivative equal to zero finds the gear ratio that gives the maximum acceleration:

$$0 = \frac{T_M\left(J_L - J_M Gr^2\right)}{\left(J_M Gr^2 + J_L\right)^2}$$

$$0 = J_L - J_M Gr^2$$

Rearranging results in:

$$Gr = \sqrt{\frac{J_L}{J_M}} \qquad \text{Equation 1}$$

This exercise basically states that for a given motor of a known torque capability, maximum load acceleration can be achieved with matched inertias. Thus, in the ideal case, it always seems advantageous to apply a gear ratio if the load to motor ratio isn't already 1 to 1. On the other hand, the application of the optimum gear ratio is not always necessary, possible, or cost effective. For example, assuming maximum acceleration is desirable, if load speeds are high and the load to motor inertia is large, applying the optimum gear ratio may result in motor speeds (as the product of the gear and load speed) well in excess of the input speed of a gear reducer, or motor itself. In this case, the highest possible ratio should be used so as not to exceed either the reducer or motor limit. Another limiting factor in applying the optimum gear ratio is the additional inertia and losses (inefficiency) of the reducer itself. Any reducer considered will have its own specific quantities of inertia, efficiency and specific gear ratio. If the inertia mismatch is already close, applying an optimum ratio may produce more losses to the system than it benefits. These issues as well as cost, need to be considered in attempting to maximize acceleration. The easiest way to consider all the variables is by using a comprehensive software sizing program. One such program is Kollmorgen's Motioneering™ that considers load variables, at the same time taking into account motor's inertia and performance data. When all these issues are considered, it is not uncommon to have a mismatch of inertia where the load inertia exceeds the motor inertia.

Practical Considerations of High Load to Motor Inertia Mismatch

When all is said and done, a large mismatch of inertia between load and motor may result. If so, why is this a problem, when is it a problem, how much mismatch is too much, and what can be done about it?

To begin to answer these questions, lets consider the problem of inertia mismatch in context of the system, not just power transfer. For servo systems to operate effectively, servo amplifiers need to be tuned to optimize the response of the system. Increasing the response of the system often involves increasing gains. Adding too much gain will lead to instability and sometimes uncontrollable oscillations. The goal is to tune the system for maximum responsiveness with the minimum of instability. Instability begins with overshoot with respect to the speed for which the motor has been given a command. A good compromise between responsiveness and stability is for the system to have critical damping and phase shift not to exceed 90°. Slightly higher phase shift has been suggested possible, however, based on variations in system requirements, mechanics and controller.

Critical damping defines an overshoot of less than 5%. The particular gains to achieve this response are based on factors such as system inertia and friction, to mention just two. Inertia is a key variable that may change in a system as a result of various factors. If inertia changes to too great an extent, the amplifier tuning may now be unacceptable.

Common tuning methods incorporated in today's servo amplifiers include Proportional/ Integral (PI), Proportional/ Integral/ Derivative (PID), Pseudo Derivative Feedback (PDF), and Pseudo Derivative feedback with Feed Forward (PDFF). Another tuning method recently introduced is Pole Placement. Each of these methods from simple to complex, have value as applied to different types of applications. Each however, has limits to keeping a system stable in the context of a varying inertia load.

To demonstrate this, a system was run in which load inertia was varied. The response of the system was measured. Two conditions were tested. The first was a system optimally tuned for motor inertia only. Then, the inertia of the system was increased by the addition of inertia wheels to the motor shaft. The resulting change in response was measured. In the second test, a system was optimally tuned for 5 times the motor inertia. Changes in the system performance were then measured as the load inertia was reduced. The results of these tests are shown below.

<u>System Components</u>

Motor -

Kollmorgen GoldLine B-202-B brushless PM servo motor having 1900 RPM maximum speed (115 VAC operation), resolver feedback, 2.44 N-m Continuous torque, 4.86 N-m Peak torque (amplifier limited), 0.0000996 Kg-m^2 rotor inertia.

Amplifier -

Kollmorgen ServoStar SR-03000 having sinusoidal current, rated 3 Amps RMS continuous, 6 Amps RMS peak, 115 to 230 VAC operation (115 VAC operation for this test). This amplifier can be tuned with PI, PDFF, or Pole Placement methods.

Power Supply -

Kollmorgen PA-0800 rated 115 1∅ or 230 3∅ VAC line operation, 0.65 kW (115 VAC, 1∅ operation).

Inertia Wheels -

Various wheels were used singly and together to derive the values shown in the test charts.

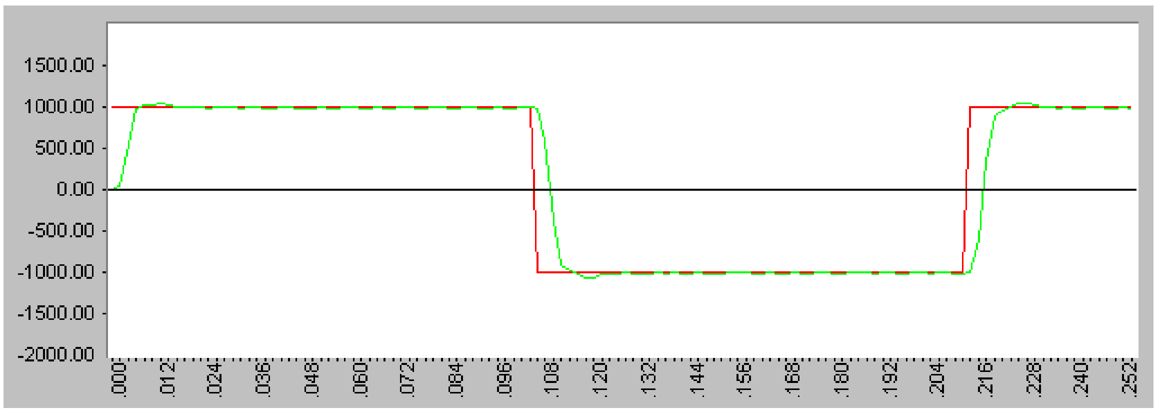
Test 1

Condition - System was tuned using Pole Placement method for tuning. Amplifier tuning parameters include: Load Inertia ratio (L.I. in %), Bandwidth (BW. in Hz.), Tracking Factor (T.F. an integer). System was tuned optimally for motor inertia only. A step command to 1000 RPM was given. The results of the tests are shown in Table 2.

Motor Inertia ($Kg\text{-}m^2$)	Load Inertia ($Kg\text{-}m^2$)	Inertia Ratio J_l/J_m	Response Plot	Overshoot (%)
0.0000996	0	0	Plot 1	5.0
0.0000996	0.0000996	1	Plot 2	16.5
0.0000996	0.0002509	2.52	Plot 3	29.5
0.0000996	0.0005017	5.04	Plot 4	38.5
0.0000996	0.001255	12.6	Plot 5	42.0
0.0000996	0.002120	21.3	Plot 6	38.0

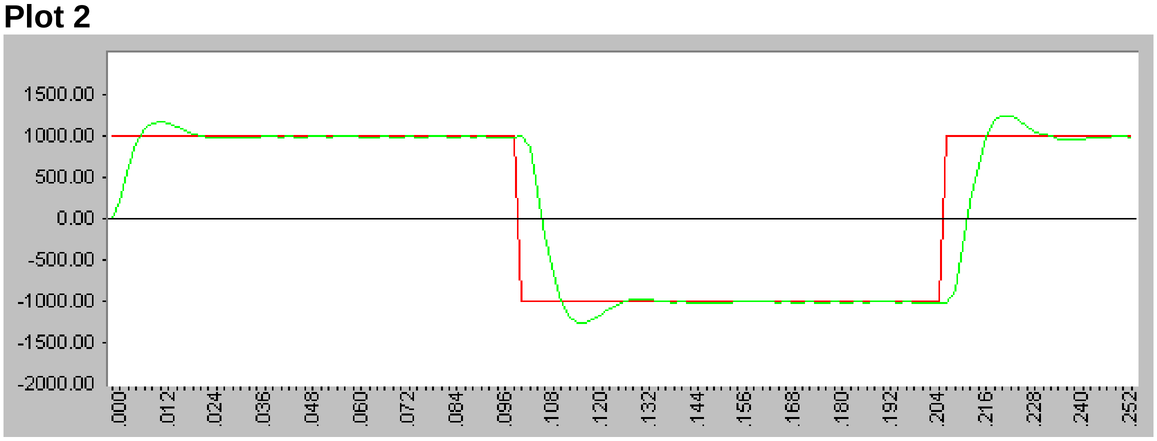
TABLE 2 - Test 1 Results

Plot 1



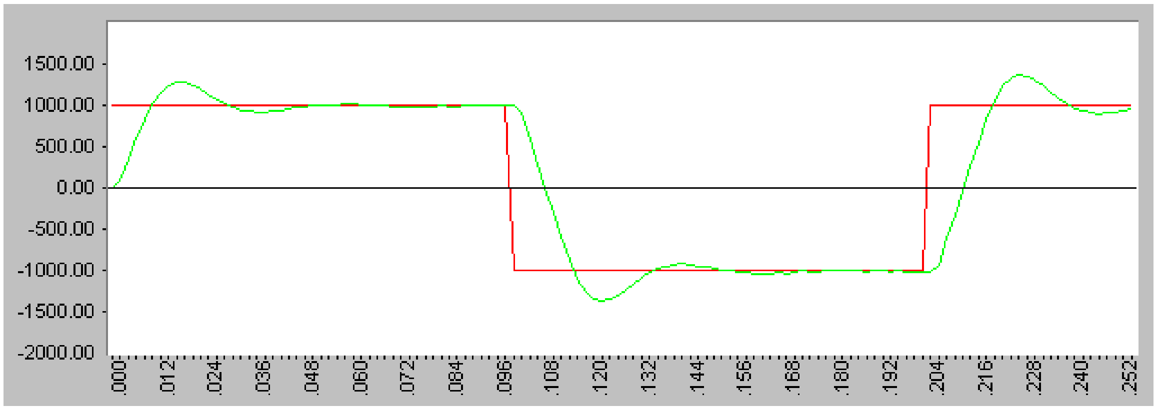
Pole placement tuning for Jm only. Amplifier tuning parameters: L.I. = 1%, BW. = 90Hz., T.F. = 100 (overshoot = 1050 rpm)

Plot 2



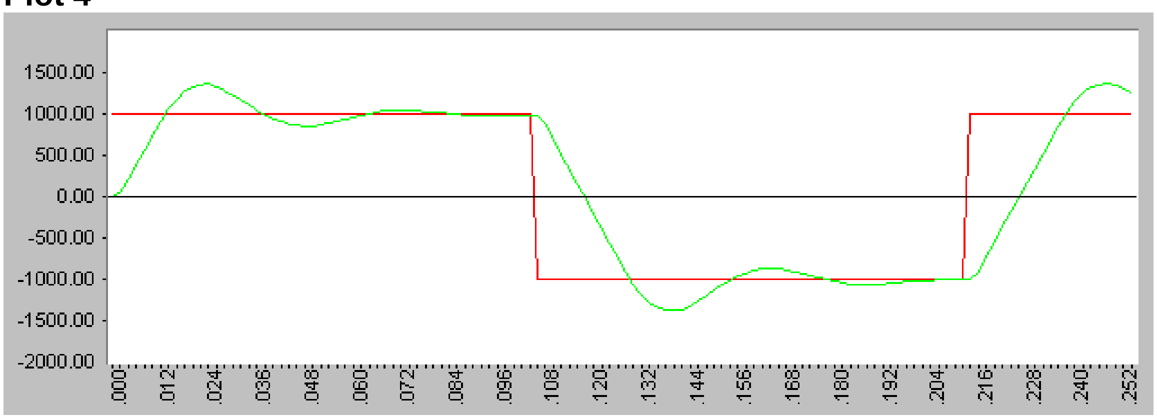
Same except J_l/J_m = 1.0 (overshoot = 1165 rpm)

Plot 3



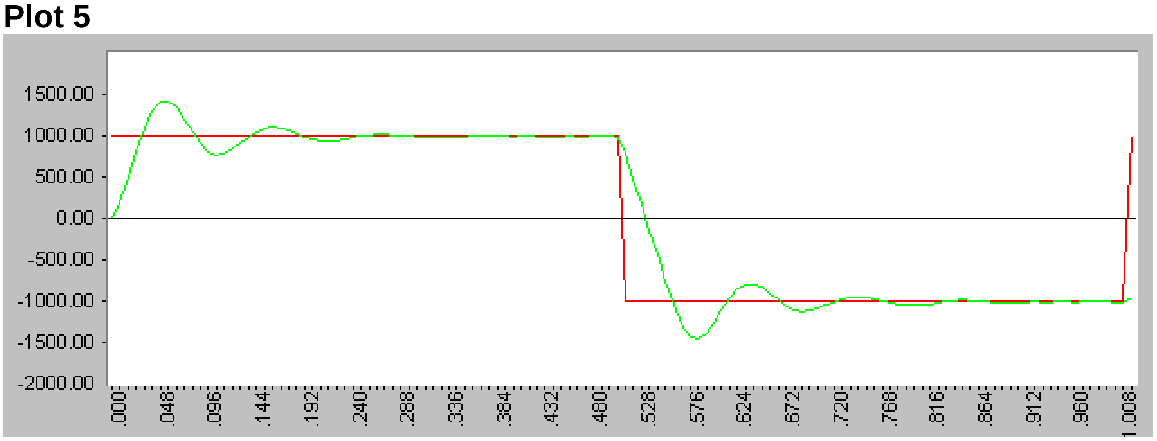
Same except J_l/J_m = 2.52 (overshoot = 1295 rpm)

Plot 4



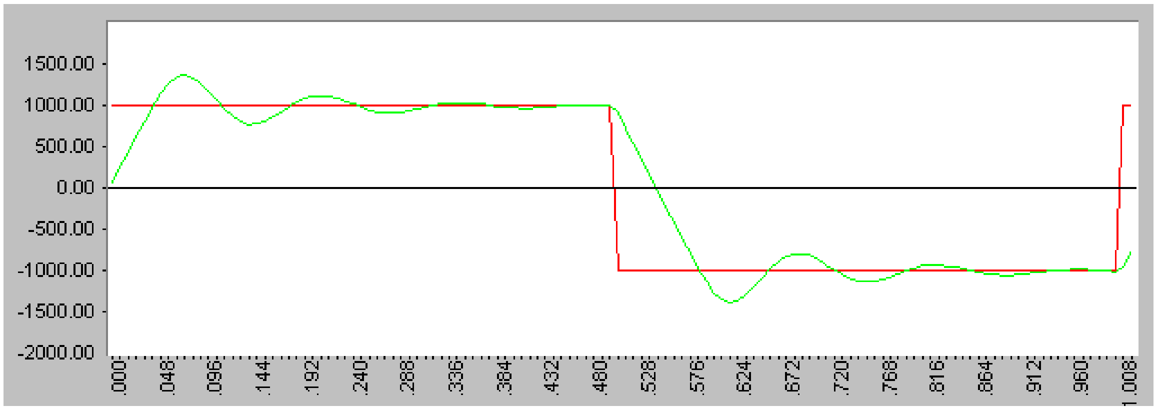
Same except J_l/J_m = 5.04 (overspeed = 1385 rpm)

Plot 5



Same except J_l/J_m = 12.6 (overshoot 1420 rpm)

Plot 6



Same except J_l/J_m = 21.3 (overshoot 1380 rpm) Freq. instability - 4.66 Hz.

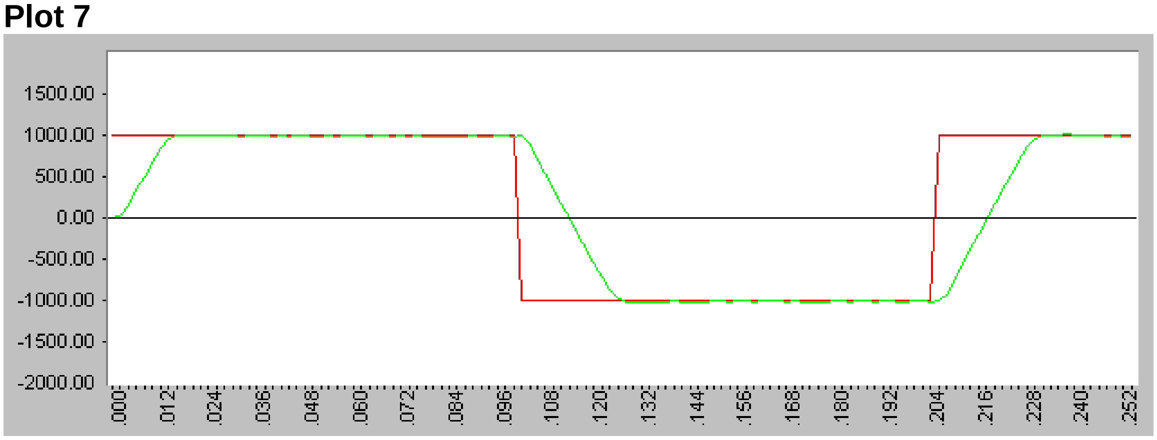
<u>Test 2</u>

Condition - System was again tuned using Pole Placement method. Amplifier tuning parameters include: Load Inertia ratio (L.I. in %), Bandwidth (BW. in Hz.), Tracking Factor (T.F. an integer). System was tuned optimally for a load inertia equal to 5x the motor inertia. A step command to 1000 RPM was given. The results of the tests are shown in Table 3.

Motor Inertia (Kg-m^2)	Load Inertia (Kg-m^2)	Inertia Ratio J_l/J_m	Response Plot	Overshoot (%)
0.0000996	0.0005017	5.04	Plot 7	0
0.0000996	0.0003505	3.52	Plot 8	0
0.0000996	0.0002509	2.52	Plot 9	0
0.0000996	0.0002305	2.31	Plot 10	N.A.

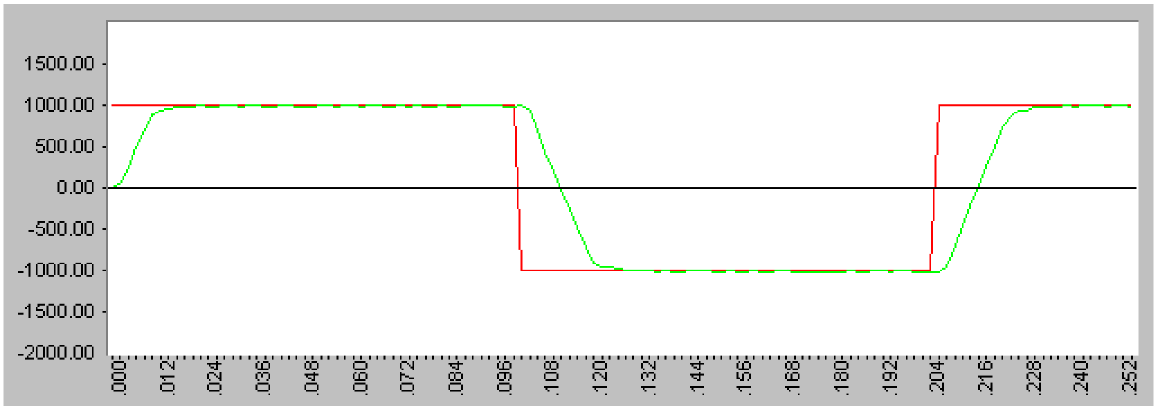
TABLE 3 - <u>Test 2 Results</u>

Plot 7



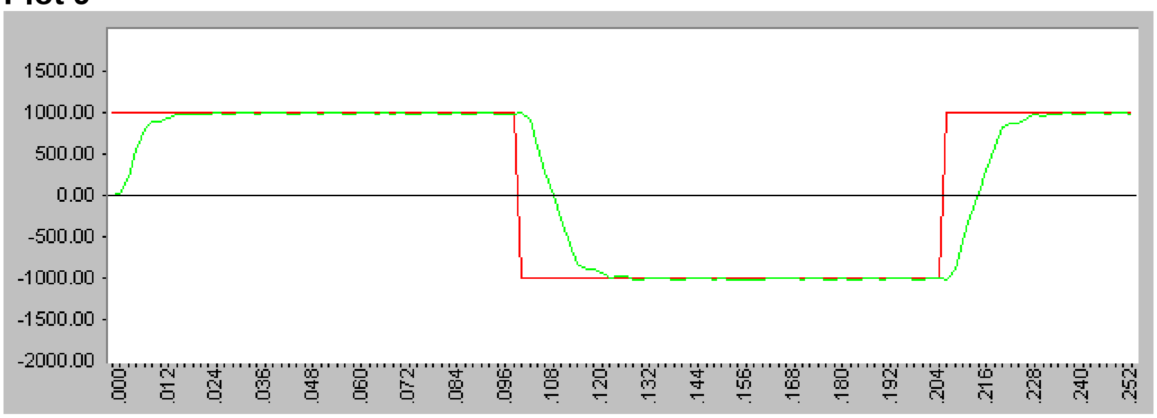
Amplifier tuning for 5.04xJm load. L..I. =432%, BW. = 90Hz., T.F. = 130

Plot 8



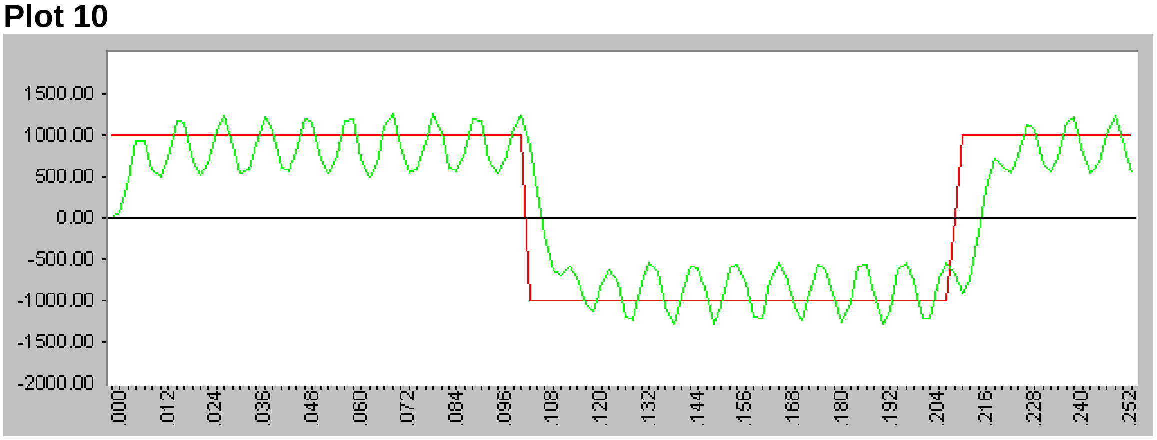
Same except J_l/J_m = 3.52

Plot 9



Same except J_l/J_m = 2.52

Plot 10



Same except J_l/J_m = 2.31 (system became unstable, Freq. instability - 100 Hz.)

To summarize the results from tests 1 and 2, two basic observations can be made. The first is that from an initial tuning, both systems could be made stable and responsive without overshoot or ringing regardless of the ratio of inertia mismatch (see next section). Secondly, once tuned, the loads responded poorly as the inertia of the system was increased or decreased significantly. For example, increasing inertia relative to the value for which it was tuned, results in the motor overshooting the command. The frequency of instability is low, and requires longer and longer settling times as the inertia increases. On the other hand, as inertia is decreased relative to a given tuning, at something less than half the inertia for which the system was tuned it became unstable at a relatively high frequency.

Why and When is Inertia Mismatch a Problem?

As has been shown, increasing or decreasing inertia significantly for a given tuned system would be considered unsatisfactory for a typical servo system. For most applications however, the load rarely changes, or if it does, generally not to the extremes noted in the tests. The effective reflected inertia does change however, in the context of lost motion (deadband) or compliance (springiness) in the system between the motor and load. This is the crux of the problem. All mechanical systems have some degree of compliance, and some systems have lost motion. Either of these, if of a sufficient quantity can result in the system responding poorly as in the test examples. The other issue of course is the ratio of load to motor inertia mismatch. For example, if the load to motor inertia ratio is 1 and the load inertia 'goes away' due to lost motion in the mechanical system, the servo loop will likely stay stable. The problem of inertia mismatch is thus mismatch in the presence of a compliant or lost motion connected load.

How Much Inertia Mismatch is Too Much?

The question really becomes, 'How much inertia mismatch is too much with regard to various types of mechanisms all having varying degrees of compliance?' This is the critical question. First, let's make some assumptions. One, the servo amplifier has sufficient range in its tuning algorithms to handle a wide range of 'system' inertia (system inertia defined as load plus motor inertia). And two, the motor and feedback device have sufficient mechanical stiffness to keep their torsional resonance frequencies well above any frequencies that would be excited by the system. With these concerns eliminated, some practical guidelines can made. Two specific case studies for direct coupled loads will be considered along with some other general guidelines for reducer coupled loads.

Direct Drive Rotary Table Study

In this example, the system consists of a Kollmorgen BMHR-6101-A brushless permanent magnet servomotor having resolver feedback driven by a ServoStar SR-10000 servo amplifier. A large rotary table was directly attached to the motor shaft. Important parameters are listed below.

<u>BMHR-6101-A Motor:</u>

Torque constant - 6.2 lb-ft/ Amp/∅ (8.41 N-m/ Amp/∅)
Peak torque - approx. 120 lb-ft (163 N-m) @ 20 Amps
Rotor inertia - 0.00852 lb-ft-s^2 (0.01155 Kg-m^2)

<u>SR-10000 Amplifier:</u>

Sinusoidal current

Continuous current - 10 Amps/∅
Peak current - 20 Amps/∅
PI, PDFF, or Pole Placement tuning options

<u>Load:</u>

Table dimensions - 39 inch (0.99 meter) diameter, 1.0 inch (25.4 mm) thick
Table material - Aluminum
Calculated inertia - 4.7lb-ft-s^2 (6.37 Kg-m^2)

This system has a load to motor inertia mismatch of 552 yet was stabilized to perform accurate 180 degree moves in 1.0 second using pole placement tuning. The system is considered noncompliant and was tuneable due to the robustness of the ServoStar amplifier.

Direct Coupled Ballscrew Study

This system consisted of a GoldLine M-413-A servomotor having resolver feedback driven by a BDS4-206J servo amplifier. A machine tool slide was coupled via a precision ballscrew to the motor shaft by a rigid coupling. Details follow.

<u>M-413-A Motor:</u>

Torque constant - 1.26 N-m/ Amp/∅
Continuous torque - 6.0 N-m
Peak torque - 15.1 N-m) @ 12 Amps
Rotor inertia - 0.00104 Kg-m^2

<u>BDS4-206J Amplifier:</u>

Sinusoidal current
Continuous current - 6 Amps/∅
Peak current - 12 Amps/∅
PDF tuning

<u>Load:</u>

Screw dimensions - 32 mm diameter, 663 mm long
Screw lead - 12 mm/rev.
Screw inertia - 0.0005412 Kg-m^2
Coupling inertia - 0.00026 Kg-m^2
Slide weight with part - 1115 Kg
Slide weight and part inertia - 0.004067 Kg-m^2
Total reflected load inertia - 0.004868 Kg-m^2

This system has a load to motor inertia ratio of 4.7. The system met the desired acceleration performance of 3.5 m/s^2 , and stability requirements. Previous to this final system selection, a lower inertia B-412 series Goldline motor was tried, yielding a load to motor inertia ratio of 10.4. Although the B series motor netted higher acceleration performance, the system was found to be too sensitive to changing machine dynamics and thus unsuitable for production equipment. In this application the screw had sufficient compliance to require a compromise between acceleration performance and machine stability due to inertia mismatch. Keep in mind that the compliance of a ballscrew increases as the ball nut moves away from the motor connected end. Historically, the goal for ballscrew loads is to keep the reflected inertia under 2 or 3 times the motor inertia. With this goal in mind, it is common to expect the peak torque requirements of the motor to be three times the continuous rating in order to meet acceleration rates typical of today's high performance machines.

Reducer Connected Loads

This discussion covers the general application of gearbox and timing belt connected loads. It needs to be stated that in the discussion of servo systems, that not just any gearbox or belt drive reducer will do. Servo systems require tight coupling with as little backlash and compliance as possible due to four quadrant operation and accuracy. For this reason, servo rated components need to be specified. Many manufacturers of these components have their own guidelines for allowable limits of load to motor inertia mismatch. For example, Harmonic Drive Systems Limited manufactures a reducer incorporating a circular spline and dynamic elliptical spline giving large reduction ratios. Their recommendation is for the reflected load inertia not to exceed 3 to 5 times the motor inertia depending on the stiffness of the load connected to the reducer output. This in general is easily accomplished based on the high ratios available. Manufacturers of planetary gear reducers make similar recommendations. Thompson Micron™ for example suggests the reflected load inertia not to exceed 4 to 10 times the motor inertia. This range is determined from motor and drive manufactures based on their experience with specific component designs, and expected performance. Timing belt connected loads also have their limits regarding reflected load inertia. These limits are not as definitive since belt type, tension, and belt length between load and motor pulleys may vary widely. For a properly sized servo rated and tensioned belt of a short span, the allowable mismatch might be as high as 5 to 1. The mismatch might be considerable less however with long spans or improper tensioning. Advancements in belt technology, like the use of kevlar impregnation is minimizing the compliant factor. Chain drives or any other mechanisms with known mechanical backlash present an obvious problem. In these cases, a load to motor inertia ratio of 1 or less is desirable. This type of system will see two different inertias, one when driving the load and another when stopped, inside the backlash.

What Can Be Done If Too Much Mismatch Exists?

From the 'Direct Drive Rotary Table' example earlier, in can be observed that a high inertia mismatch is not necessarily a problem for a very rigid system. What makes inertia mismatch a problem, as previously stated is compliance or lost motion in the drive mechanism, or the inability of the controller to handle high system inertia. The first consideration in dealing with high mismatches of inertia for compliant connected loads is to apply appropriate reduction to minimize the mismatch. The second is to stiffen the mechanical system. If these efforts fail to sufficiently reduce the mismatch, the alternative is to increase inertia on the motor end of the drive train to minimize the mismatch. This is typically done with the use of increased inertia motors. Oversizing motors is an expensive solution unless higher inertia motors of equivalent torque ratings are available at comparable prices. Most large suppliers of servo motors offer such a line of motors. Kollmorgen for example, offers such motors ranging from roughly 3 to 7 times the inertia of an equivalent torque rated motor of the low inertia type. An example of this type is the motor chosen in the case study ‘Direct Coupled Ballscrew’. The M series motor chosen has an inertia 2.2 times the inertia of the B series type also considered, and provided the necessary compromise between system stability and acceleration performance.

Conclusion

The concern for mismatch of inertia between motor and load involves many factors when considering high performance servo systems. Low inertia motors for example, will minimize total system inertia allowing higher acceleration and bandwidth. If high load to motor inertia mismatch exists, however, it can result in load instabilities for compliant or lost motion connected systems. Mismatch can be minimized by the use of reducers while maximizing acceleration performance. Optimizing reduction for a matched load to motor inertia ratio however, may not be cost effective or possible. Higher inertia motors are sometimes the best alternative to provide a tradeoff between maximum dynamic performance and stability of the servo system. Manufacturers of servo products offer varying suggestions for allowable inertia mismatch based on known limitations of their components, rules which should be adhered to when designing systems. Ongoing improvements in the control algorithm design and mechanical systems are however, making possible more accurate and stable servos in our imperfect world.

Acknowledgments

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